Understanding Math

Concept: Solving Inequalities

Name:

- **You should have completed Equations Outline A for Topic 7: Solving Inequalities before beginning this handout.**

COMPUTER COMPONENT

Instructions: Select the computer program *Understanding Equations* (Neufeld)
Follow the instructions to the Main Menu.
Select *Solving Inequalities* from the Main Menu.

*Notice: You will need to use the **Jump To** feature of the program (found on the top left of your screen) in order to get to the section where you left off.*

Work through all sections of this topic **in order**:

- *Graphing Linear inequalities in Two Variables*
- *Solving Systems of Linear Inequalities by Graphing*
- *Linear Programming*
- *Practice Questions*

Additional Required Materials: Pencil Crayons

As you work through the computer exercises, make your notes in the **NOTES** section of this page.

When you reach the end of the section *Practice Questions* on the computer, move on to the **OFF COMPUTER EXERCISES** below**.**

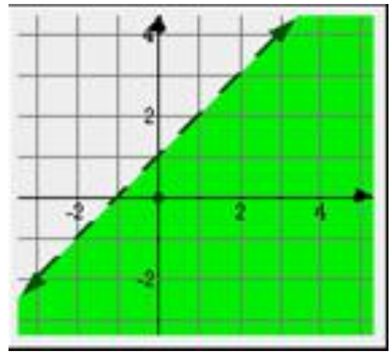
NOTES:

Graphing Linear Inequalities in Two Variables

The solution is not a **point** on the graph.

The solution is a **region** of the graph.

The graph displays the linear inequality $y < x + 1$



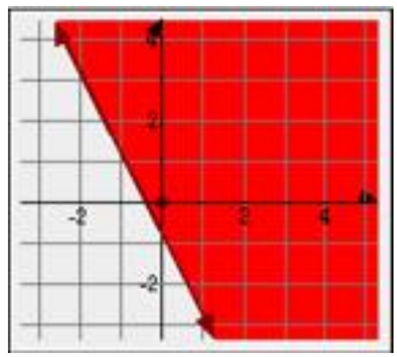
The points **on** the boundary line (**dotted** line)

represent all points where y = x+1.

The points **below** the boundary line (**dotted** line)

represent all points where y < x +1.

The graph displays the linear inequality $y \geq -2x - 1$



The points **on** the boundary line (**solid** line)

represent all points where y = -2 x - 1

The points **above** the boundary line (**solid** line)

represent all points where y **>** -2 x - 1.

Checking a point on either side of the boundary line will help determine if that region is in the solution.

Steps to graphing Linear Inequalities in Two Variables: (*Fill in the blanks*)

Step 1: **Graph** the boundary line.

Step 2: Determine if the boundary line is **dotted** or **solid.**

Step 3: Pick a point on either side of the line and check if that point is in the solution.

Example: $y < x + 1$

Check (0, 0) $y < x + 1$

$\mathbf{0} < \mathbf{0} + 1$

$\mathbf{0} < \mathbf{1}$ **True, (0, 0) is in the solution region.**

If it is true then (0, 0) is part of the solution

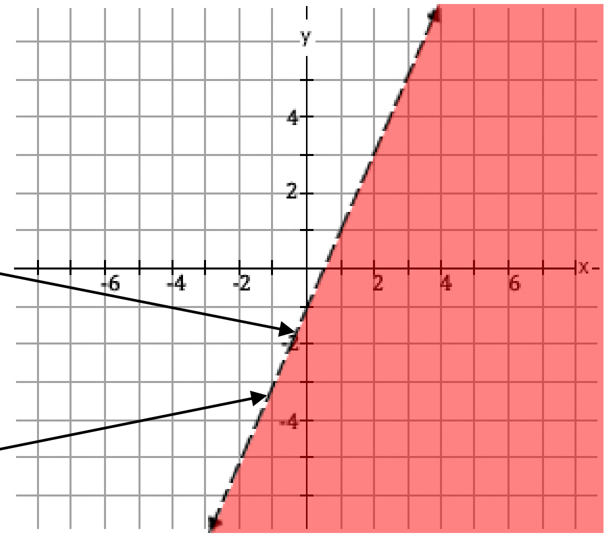
The graph of the linear inequality $y < 2x - 1$ is shown.

(a) Color the region covering points that make the inequality true.

(b) Indicate the boundary line.

(c) Write the equation for the boundary line.

$y = 2x - 1$



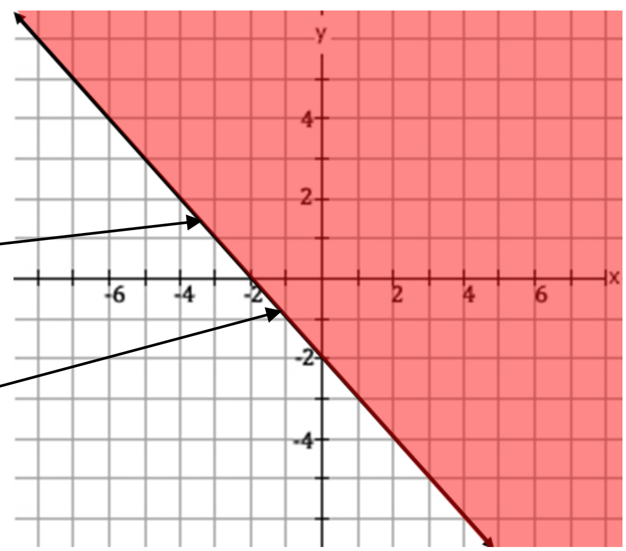
The graph of the linear inequality $y \geq -x - 2$ is shown.

(a) Color the region covering points that make the inequality true.

(b) Indicate the boundary line.

(c) Write the equation for the boundary line.

$y = -x - 2$



Explain why some boundary lines are dotted and others are solid.

(Answer may vary)
Key ideas:

- **Dotted boundary lines show that the points on the boundary line <u>are not included</u> in the solution.**
- **Inequalities with $<$ or $>$ symbols will have graphs that have dotted boundary lines.**
- **Solid boundary lines show that the points on the boundary line <u>are included</u> in the solution.**
- **Inequalities with $\leq$ or $\geq$ symbols will have graphs that have solid boundary lines.**

Solving Systems of Linear Inequalities by Graphing

- Graph each inequality **boundary line**

- The solution to the system will be the **shading** where the shadings from each inequality **overlap** one another.

Linear Programming:

> *"The food processing industry is perhaps the second most active user of linear programming, where it was first used to determine shipping of ketchup from a few plants to many warehouse."*
>
> *books.google.ca/books?isbn=0387948333*

Linear Programming is a method used to manage **resources** and **time.**

- Inequalities are used as **constraints** that **limit** the business **activity.**

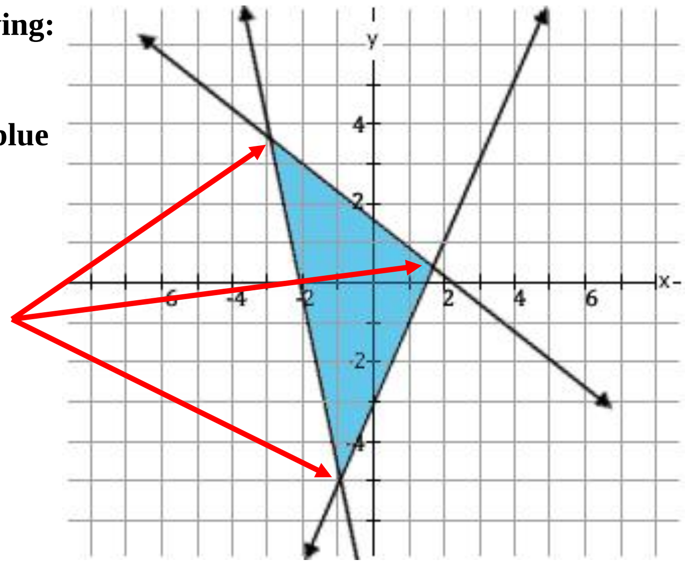
- The **solution** of the **system** of constraints is called the **feasibility** **region.**

- The **vertices** of the **feasibility** region are possible **maximum** and **minimum** values.

In the following graph indicate the following:

(a) Feasible region
The feasible region is shaded blue

(b) Vertices of the feasible region



OFF COMPUTER EXERCISES

1. Graph the following inequalities.

(a) $y > 2x - 4$

Graph the boundary line using $y = 2x - 4$

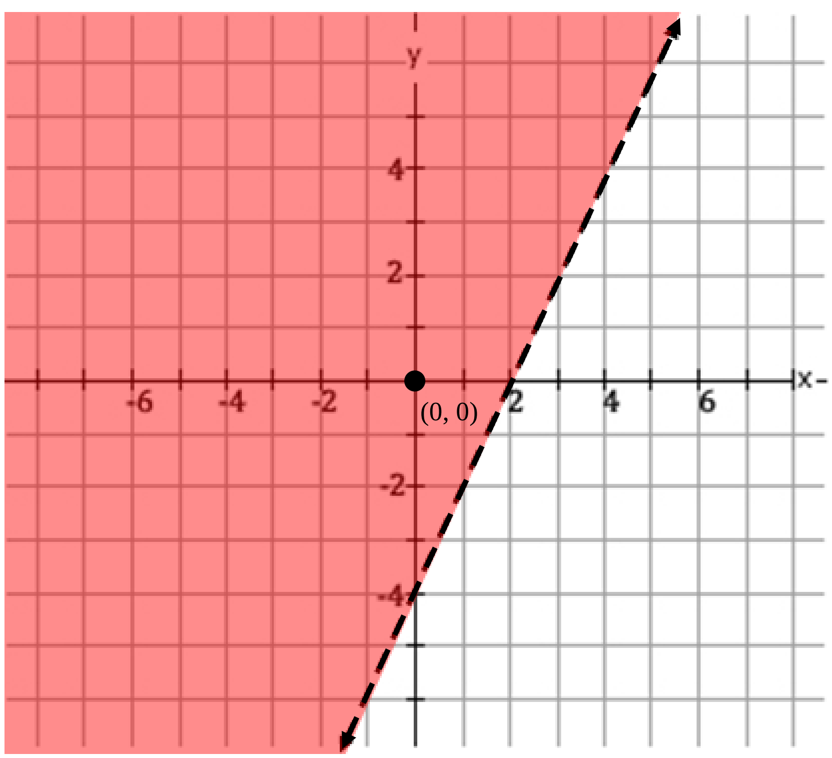
y-intercept is (0, -4)
Slope of the line is 2

Since $y > 2x - 4$**, the boundary line is dotted.**

Evaluate the inequality using test point (0, 0)

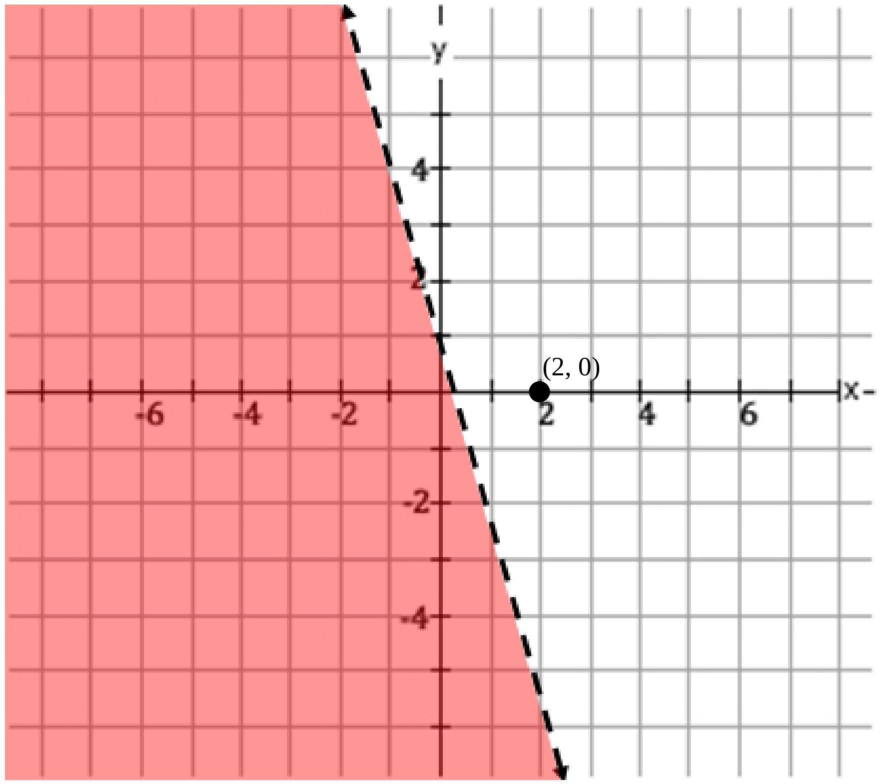
$\therefore$ $0 > 0 - 4$
$0 > -4$, **True**

$\therefore$ **(0, 0) is in the shaded region**



(b) $y < -3x + 1$

Graph the boundary line using $y = -3x + 1$



y-intercept is (0, 1)
Slope of the line is -3

Since $y < -3x + 1$, the boundary line is dotted.

Evaluate the inequality using test point (2, 0)

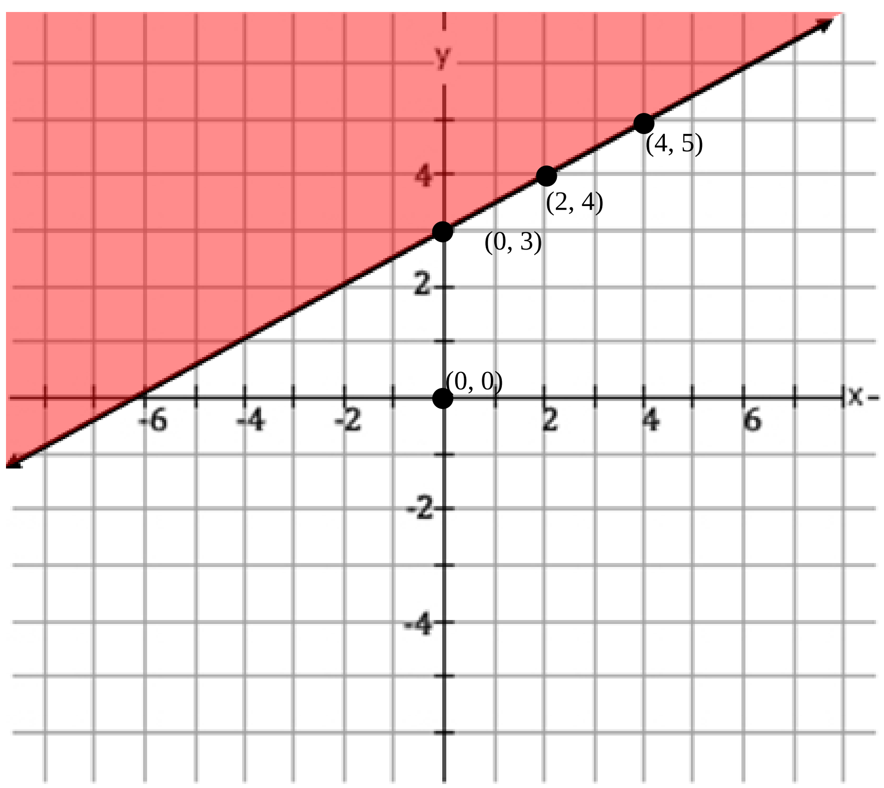
∴ **$0 < -6 + 1$**
$0 < -5$, False

∴ **(2, 0) is NOT in the shaded region.**

(c) $y \geq \frac{1}{2}x + 3$

Graph the boundary line using $y = \frac{1}{2}x + 3$

x	y
4	5
2	4
0	3



Since $y \geq \frac{1}{2}x + 3$, the boundary line is solid.

Evaluate the inequality using test point (0, 0)

∴ **$0 \geq 0 + 3$**
$0 \geq 3$, False

∴ **(0, 0) is NOT in the shaded region.**

2. Solve each system of inequalities. (*Hint: Find the feasible region.*)

(a) $\begin{Bmatrix} y \geq 2x - 4 \\ y < -x + 5 \end{Bmatrix}$

Using $y = 2x - 4$

y-intercept is (0, -4)
Slope of the line is 2

Since $y \geq 2x - 4$, the boundary line is solid.

Using $y = -x + 5$

y-intercept is (0, 5)
Slope of the line is -1

Since $y < -x + 5$, the boundary line is dotted.

Substitute test point (0, 0) into $y \geq 2x - 4$

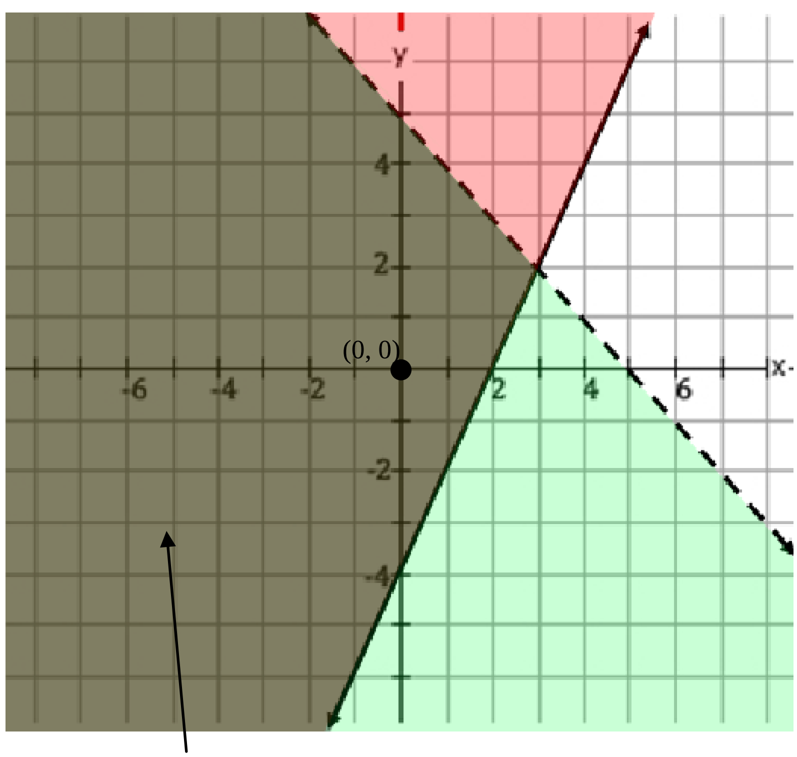
∴ $0 \geq 0 - 4$
$0 \geq -4$, True

∴ (0, 0) is in the shaded region for $y \geq 2x - 4$.

Substitute test point (0, 0) into $y < -x + 5$

∴ $0 < 0 + 5$
$0 < 5$, True

∴ (0, 0) is in the shaded region for $y < -x + 5$



The area where regions overlap is the feasible region.

(b) $$\begin{cases} y > \frac{1}{2}x - 3 \\ y \le 2x - 5 \end{cases}$$

Using $y = \frac{1}{2}x - 3$

y-intercept is (0, -3)
Slope of the line is $\frac{1}{2}$

Since $y > \frac{1}{2}x - 3$, the boundary line is dotted.

Using $y = 2x - 5$

y-intercept is (0, -5)
Slope of the line is 2

Since $y \le 2x - 5$, the boundary line is solid.

Substitute test point (0, 0) into $y > \frac{1}{2}x - 3$

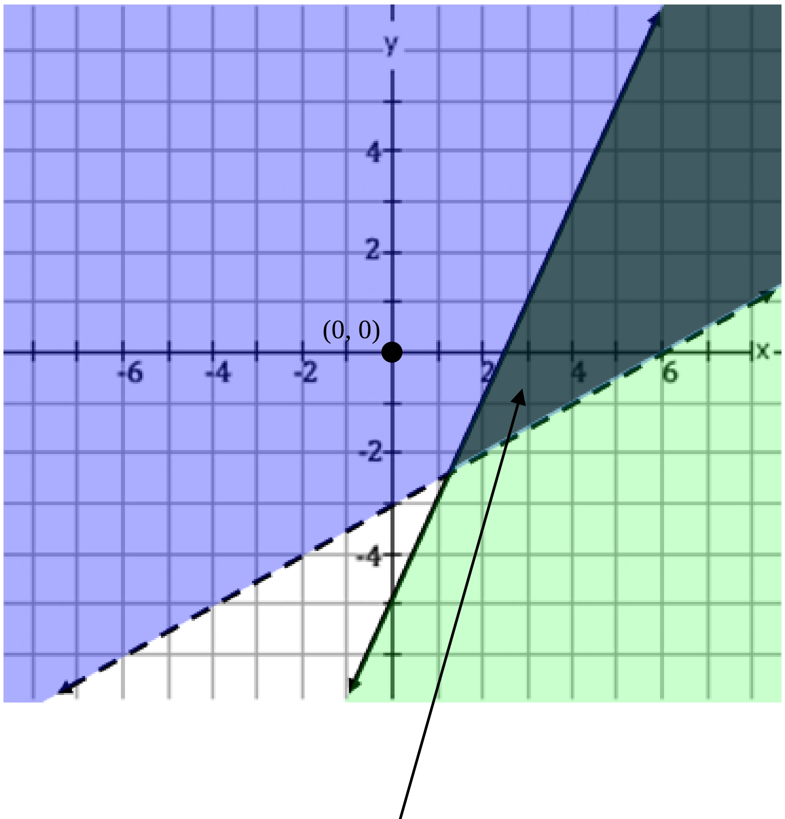
∴ $0 > 0 - 3$
$0 > -3$, True

∴ (0, 0) is in the shaded region for $y > \frac{1}{2}x - 3$.

Substitute test point (0, 0) into $y \le 2x - 5$

∴ $0 \le 0 - 5$
$0 \le -5$, False

∴ (0, 0) is NOT in the shaded region for $y \le 2x - 5$



The area where regions overlap is the feasible region.

(c) $$\left\{ \begin{array}{l} y > 1 \\ y < x + 2 \end{array} \right\}$$

Using $y = 1$

y-intercept is (0, 1)
Slope of the line is 0. Therefore, for any value of x, $y = 1$

Since $y > 1$, the boundary line is dotted.

Using $y = x + 2$

y-intercept is (0, 2)
Slope of the line is 1

Since $y \leq x + 2$, the boundary line is solid.

Substitute test point (0, 0) into
$y > 1$

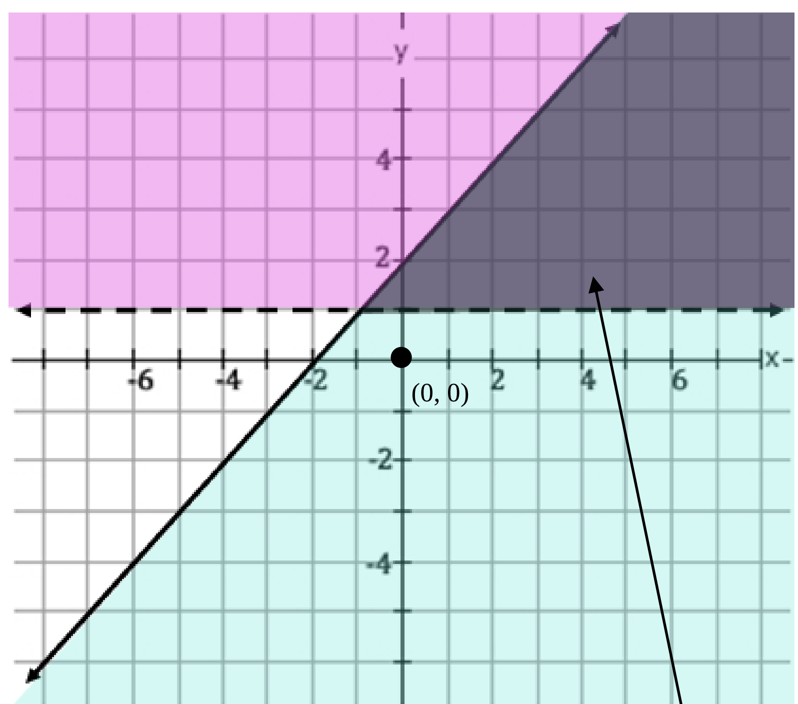
∴ $0 > 1$, False

∴ (0, 0) is NOT in the shaded region $y > 1$.

Substitute test point (0, 0) into
$y < x + 2$

∴ $0 < 0 + 2$
$0 < 2$, True

∴ (0, 0) is in the shaded region for $y < x + 2$.



The area where regions overlap is the feasible region.

3. A tire manufacturer makes two types of tires; regular and winter. A maximum of 200 regular tires and 125 winter tires can be manufactured in one day. The finishing machine can only handle up to 250 tires in one day. The profit on each regular tire is $25 and on each winter tire is $30. *How many of each type should be made in order to maximize profits?*

Solution:

Let the number of regular tires be represented by x and the number of winter tires be represented by y.

We must now determine our constraints.

$x \leq$ **<u>200</u>**

$y \leq$ **<u>125</u>**

$(x + y) \leq$ **<u>250</u>** (*Hint: The finishing machine*)

Graph each of the three constraints above.

Don't forget to shade the appropriate areas in order to find the feasible region.

- You should find that your feasible region is formed by 5 vertices:

(0, 0) , (0, **<u>125</u>**) , (**<u>200</u>** , 0) , (**<u>200</u>** ,50) , (125, **<u>125</u>**)

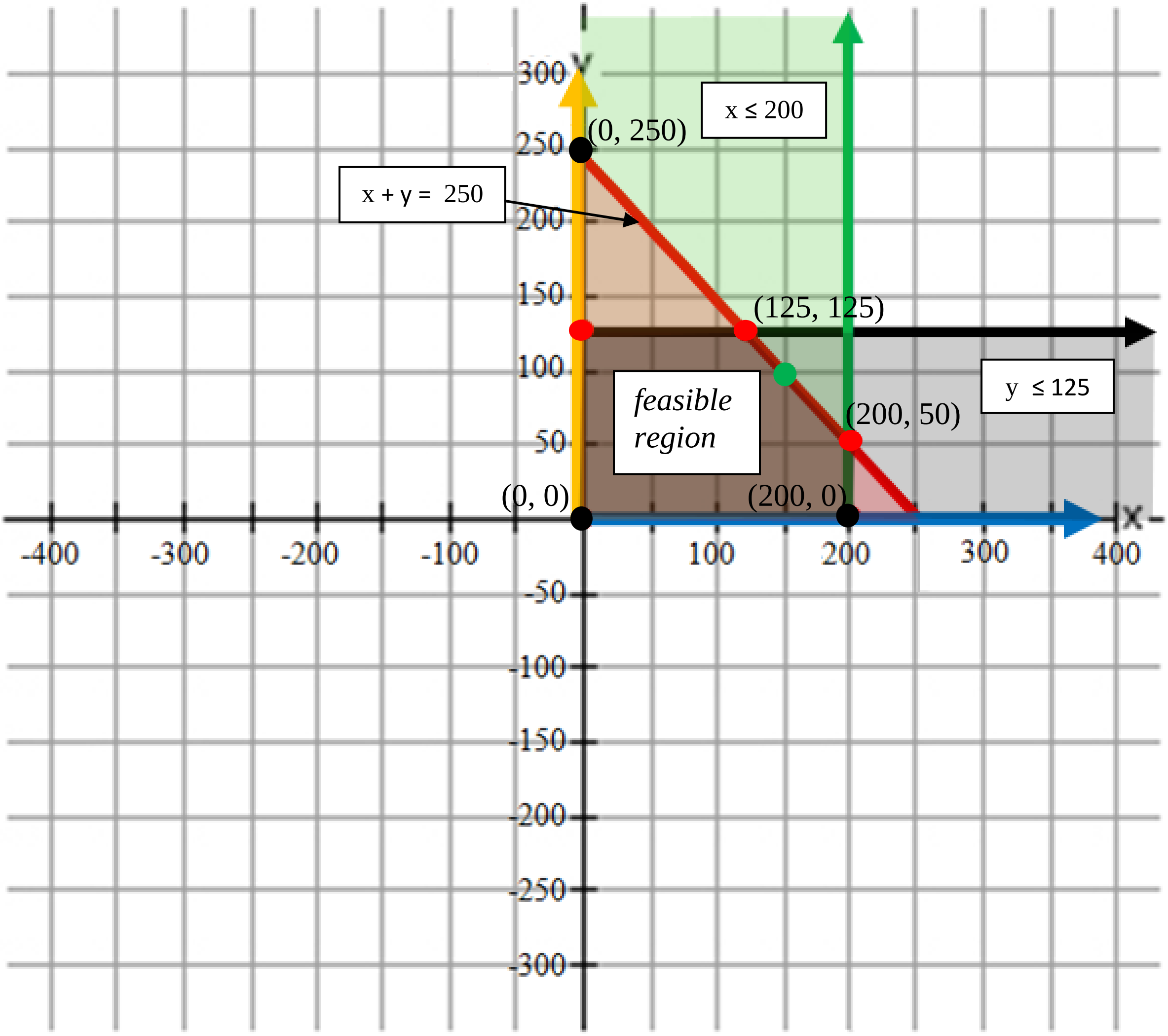
For maximum production, the finishing machine must be kept running at full capacity.

x	y
50	200
100	150
125	125
150	100
200	50

$\therefore \quad x + y = 250$

$-x) \quad x + y - x = 250 - x$

$y = 250 - x$



Notice that you cannot make a negative number of tires therefore both x and y must be equal to or greater than 0. $y \geq 0$, $x \geq 0$

Remembering that our goal is to find what combination will produce the maximum profit, we now want to substitute each set of vertices into the profit equation

Let P represent the profit:

P = 25x + 30y (This information was given in the question)

At (0, 125) P = 25x + 30y
= 25(0) + 30(125)
= 3750

At (125, 125) P = 25x + 30y
= 25(125) + 30(125)
= 3125 + 3750
= 6875

At (200, 0) P = 25x + 30y
= 25(200) + 30(0)
= 5000

At (200, 50) P = 25x + 30y
= 25(200) + 30(50)
= 5000 + 1500
= 6500

After all 5 substitutions have been made, *what combination of tire can you conclude will bring the company maximum profit? Justify your answer.*

i) The solution lies on the boundary of the feasibility region since any point within the region corresponds to the finishing machine being partly idle, that is, less than 250 tires being processed a day.

ii) From the graph, when $x < 125$, winter tire production would be at the maximum possible per day (125) and the finishing machine would be partly idle.
$125 + x < 250$ when $x<125$

iii) From the graph, when $x > 125$, the number of winter tires that can be finished in a day decreases because of the stated capability of finishing a maximum of 250 per day, for example, if 200 standard tires are finished in a day, only 50 winter tires can be finished even though 125 can be produced.

From the profit calculations above and the logic stated, the maximum profit occurs when 125 of each tire is produced and finished. If x is less than 125 or greater than 125 the profit goes down from 6875.