Understanding Math

Concept: Solving Linear Systems

Name:

- **You should have completed Equations Outline A for Topic 6: Solving Linear Systems before beginning this handout.**

COMPUTER COMPONENT

Instructions: Select the computer program *Understanding Equations* (Neufeld)
Follow the instructions to the Main Menu.
Select *Solving Linear Systems* from the Main Menu.

*Notice: You will need to use the **Jump To** feature of the program (found on the top left of your screen) in order to get to the section where you left off.*

Work through all sections of this topic **in order**:

- *Solve a Linear System by Comparison*
- *Solve Problems Using Linear Systems*
- *Practice Questions*

As you work through the computer exercises, make your notes in the **NOTES** section of this page.

When you reach the end of the section *Practice Questions* on the computer, move on to the **OFF COMPUTER EXERCISES** below**.**

NOTES:

Solve a Linear System by Comparison (*Intersecting Lines*)

Step	Example
1. **Isolate** one of the ***variables*** for **each** equation. (*We choose y*.)	$2x - y + 3 = 0$ $x - y - 1 = 0$ $y = \underline{2}x + \mathbf{3}$ (1) $y = \mathbf{1}x - \mathbf{1}$ (2)

2. For the **<u>point of intersection</u>** , **y** of (1) = **y** of (2)	(1) (2) $\mathbf{2}x + \mathbf{3} = \mathbf{1}x - \mathbf{1}$
3. Solve for **<u>one</u>** variable. *(**x** in this case)*	$\mathbf{2}x + \mathbf{3} = \mathbf{1}x - \mathbf{1}$ $\mathbf{2}x - \mathbf{1}x = \mathbf{-1} - \mathbf{3}$ $\mathbf{1}x = \mathbf{-4}$ $x = \mathbf{-4}$
4. Substitute x = **-4** into one of the equations to solve for **y**. ***Note: In this case substitute x = -4 into equation (1)***	$y = 2x + 3$ (1) $y = 2(\mathbf{-4}) + 3$ $y = \mathbf{-5}$ Common point is (**-4, -5**)
5. **<u>Check</u>** the solution in each **<u>original</u>** equation. ***Note: In this case substitute x = -4 and y = -5 into equation (1)***	$2x - y + 3 = 0$ (1) $x - y - 1 = 0$ (2) For (1) ***Substitute x = -4 and y = -5 into equation (1)*** L.S. $= 2x - y + 3$ $= 2(\underline{-4}) - (\underline{-5}) + 3$ $= \underline{\mathbf{0}}$ R.S. = 0 Same, L.S. = R.S.

Note: In this case substitute x = -4 and y = -5 into equation (2)	For (2) ***Substitute x = -4 and y = -5 into equation (2)*** L.S. = x – y - 1 = (**<u>-4</u>**) - (**<u>-5</u>**) - 1 = **<u>0</u>** R.S. = 0 Same, L.S. = R.S. ***Since L.S. = R.S. for both equations then the solution x = -4, y = -5 is correct.***

Solving a Linear System by Comparison (Intersecting Lines Involving Fractions)

- For each equation, **<u>clear</u>** the **<u>fractions</u>** from the equation.

  (**<u>multiply</u>** each term by a **<u>common</u>** **<u>denominator</u>**.)

  Once we have **<u>cleared the fraction</u>** and **<u>expanded</u>** the bracket, you now have equations with which you can continue to solve using the above steps for solving a linear System by Comparison (Intersecting Lines).

Solving a Linear System by Comparison (Parallel Lines)

- Parallel lines do not intersect. Therefore **<u>y</u>** of (1) cannot **<u>equal</u>** **<u>y</u>** of (2).

  It is **<u>not</u>** possible to solve for **<u>x</u>** (or for **<u>y</u>**), and these types of Linear Systems have **<u>no</u>** **<u>solution</u>**.

Solving a Linear System by Comparison (Coincidental Lines)

- Coincidental Lines are IDENTICAL. **<u>All</u>** points on line **<u>one (1)</u>** are also on line **<u>two (2)</u>** . This Linear System has an **<u>infinite</u>** number of **<u>solutions.</u>**

OFF COMPUTER EXERCISES

1. Solve the following linear systems by comparison.

(a) $y = 2x + 3$
$y = x - 6$

Isolate Variables
Note: both equations are already expresses in terms of y.

$$y = 2x + 3 \quad (1)$$
$$y = x - 6 \quad (2)$$

At the point of Intersection y in equation (1) = y in equation (2)

$$2x + 3 = x - 6$$
$$\text{-x)} \quad 2x - x + 3 = x - 6 - x$$
$$\text{-3)} \quad x + 3 - 3 = -6 - 3$$
$$x = -9$$

Substitute x = -9 into equation (2)

$$y = x - 6 \quad (2)$$
$$y = -9 - 6$$
$$y = -15$$

The Solution Point is (-9, -15)

Check Equation (1) $y = 2x + 3$

L.S. $= y$
$= -15$

R.S. $= 2x + 3$
$= 2(-9) + 3$
$= -18 + 3$
$= -15$

Check Equation (2) $y = x - 6$

L.S. $= y$
$= -15$

R.S. $= x - 6$
$= (-9) - 6$
$= -15$

Since L.S. = R.S. for both equations, the solution x = -9, y = -15 is correct.

(b) $x + y - 4 = 0$
$2x + y + 1 = 0$

Isolate Variables
Note: both equations are already expresses in terms of y.

$$y = -x + 4 \quad (1)$$
$$y = -2x - 1 \quad (2)$$

At the point of Intersection y in equation (1) = y in equation (2)

$$-x + 4 = -2x - 1$$
$$+2x) \quad -x + 2x + 4 = -2x + 2x - 1$$
$$-4) \quad x + 4 - 4 = -1 - 4$$
$$x = -5$$

Substitute x = -5 into equation (2)

$$y = -x + 4 \quad (2)$$
$$y = -(-5) + 4$$
$$y = 5 + 4$$
$$y = 9$$

The Solution Point is (-5, 9)

Check Equation (1) $x + y - 4 = 0$

L.S. $= x + y - 4$ **R.S.** $= 0$
$= -5 + 9 - 4$
$= 0$

Check Equation (2) $2x + y + 1 = 0$

L.S. $= 2x + y + 1$ **R.S.** $= 0$
$= 2(-5) + 9 + 1$
$= 0$

Since L.S. = R.S. for both equations, the solution x = -5, y = -9 is correct.

(c) $5x + 2y - 8 = 0$
$2x + 4y + 8 = 0$

Isolate Variables

Isolate y

$$y = \frac{-5}{2}x + 4 \qquad (1)$$

$$y = \frac{-1}{2}x - 2 \qquad (2)$$

At the point of Intersection y in equation (1) = y in equation (2)

Clear the fraction

$$\frac{-5}{2}x + 4 = \frac{-1}{2}x - 2$$

$$\times 2)\quad -5x + 8 = -x - 4$$

$$+x)\quad -4x + 8 = -4$$

$$-8)\quad -4x = -12$$

$$\div -4)\quad x = 3$$

Substitute x = 3 into equation (1)

$$y = \frac{-5x}{2} + 4 \qquad (1)$$

$$y = \frac{-5}{2}(3) + 4$$

$$y = \frac{-15}{2} + \frac{8}{2}$$

$$y = \frac{-7}{2}$$

The Solution Point is $(3, \frac{-7}{2})$

Check Equation (1) $5x + 2y - 8 = 0$

L.S. $= 5x + 2y - 8$ **R.S.** $= 0$

$= 5(3) + 2(\frac{-7}{2}) - 8$

$= 15 - 7 - 8$

$= 0$

Check Equation (2) $2x + 4y + 8 = 0$

L.S. $= 2x + 4y + 8$ **R.S.** $= 0$

$= 2(3) + 4(\frac{-7}{2}) + 8$

$= 6 - 14 + 8$

$= 0$

Since L.S. = R.S. for both equations, the solution x = 3, y = $\frac{-7}{2}$ is correct.

(d) $3x + 2y - 5 = 0$ **(1)**
$4x + 3y - 2 = 0$ **(2)**

Isolate Variables

Isolate y

$$y = \frac{-3}{2}x + \frac{5}{2} \qquad (3)$$

$$y = \frac{-4}{3}x + \frac{2}{3} \qquad (4)$$

At the point of Intersection y in equation (3) = y in equation (4)

$$\frac{-3}{2}x + \frac{5}{2} = \frac{-4}{3}x + \frac{2}{3}$$

Clear the fraction

$$\begin{aligned} \times 6) \quad 9x - 15 &= 8x - 4 \\ -8x) \quad x - 15 &= -4 \\ +15) \quad x &= 11 \end{aligned}$$

Substitute x = 11 into equation (1)

$$\begin{aligned} 3x + 2y - 5 &= 0 \qquad (1) \\ 3(11) + 2y - 5 &= 0 \\ 33 + 2y - 5 &= 0 \\ 2y + 28 &= 0 \\ -28) \quad 2y &= -28 \\ \div 2) \quad y &= -14 \end{aligned}$$

The Solution Point is (11, -14)

Check Equation (1) $3x + 2y - 5 = 0$

L.S. $= 3x + 2y - 5$ **R.S.** $= 0$
$= 3(11) + 2(-14) - 5$
$= 33 - 28 - 5$
$= 0$

Check Equation (2) $4x + 3y - 2 = 0$

L.S. $= 4x + 3y - 2$ **R.S.** $= 0$
$= 4(11) + 3(-14) - 2$
$= 44 - 42 - 2$
$= 0$

Since L.S. = R.S. for both equations, the solution x = 11, y = -14 is correct.

2. The local fair charges $6.00 for admission, plus $0.50 for every ride ticket you buy.
   The neighboring town's fair offers free admission, but charges $1.00 for every ride ticket.
   When is the local fair the better deal?

a) Create a table of values for each fair, let the number of rides be N and the cost be C.

Local Fair cost: C = 6.00 + 0.50N

N Number of Rides	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
C Cost ($) of fair + rides	**6.50**	**7.00**	**7.50**	**8.00**	**8.50**	**9.00**	**9.50**	**10.00**	**10.50**	**11.00**	**11.50**	**12.00**	**12.50**

Neighboring Town's Fair cost: C = 1N

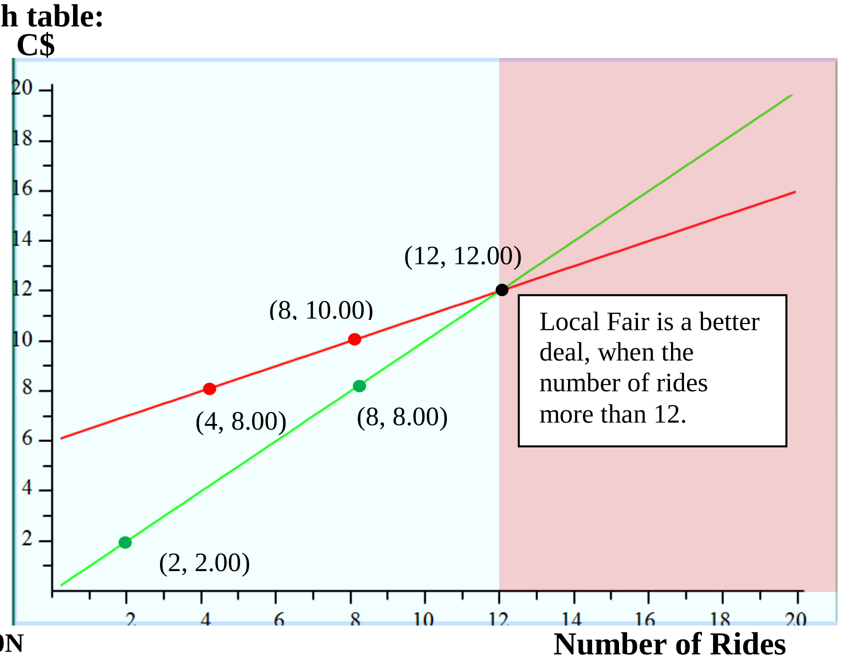
N Number of Rides	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
C Cost ($) of fair + rides	**1.00**	**2.00**	**3.00**	**4.00**	**5.00**	**6.00**	**7.00**	**8.00**	**9.00**	**10.00**	**11.00**	**12.00**	**13.00**

b) Create the equation for each table:

C = 6.00 + 0.50N (1)
C = 1N (2)

c) Graph each line:

d) Point of Intersection
(12, 12.00)



e) Solve the two equations by comparison

1N	**=**	**6.00+0.50N**
1N–0.50N	**=**	**6.00+0.50N-0.50N**
0.50N	**=**	**6.00**
÷0.50) N	**=**	**12**

f) Substitute N = 12 into equation (2)
C = 1N
C= 12.00

When is the local fair the better deal?
The local fair is better deal when the number of rides (N) is greater than 12.

3. The cost to rent a car with Company A is \$25 per day plus \$0.15 per km driven.
The cost to rent a car with Company B is \$30 per day plus \$0.10 per km driven.
Under what circumstances is Company A the better company to rent with?

a) Create a table of values for each company, let the number of km be N and the cost be C.

Company A cost: C = 25 + 0.15N

N Number of km	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**120**
C Cost (\$) In dollars	**25**	**26.50**	**28.00**	**29.50**	**31.00**	**32.50**	**34.00**	**35.50**	**37.00**	**38.50**	**40.00**	**41.50**

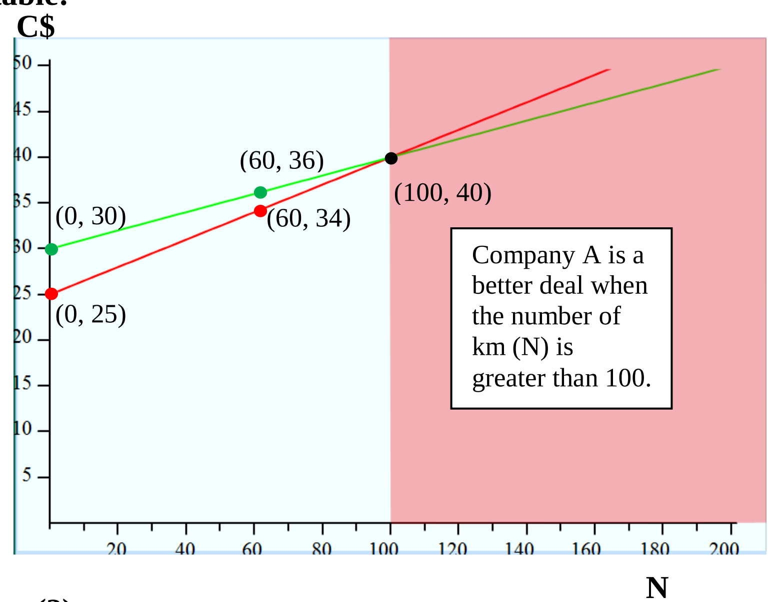
Company A cost: C = 30 + 0.10N

N Number of km	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**120**
C Cost (\$) In dollars	**30**	**31.00**	**32.00**	**33.00**	**34.00**	**35.00**	**36.00**	**37.00**	**38.00**	**39.00**	**40.00**	**41.00**

b) Create the equation for each table:

C = 25 + 0.15N (1)

C = 30 + 0.10N (2)



c) Graph each line:

d) Point of Intersection

(100, 40)

e) Solve the two equations by comparison

	25 + 0.15N	**=**	**30 + 0.10N**
-0.10N)	**25 + 0.05N**	**=**	**30**
-25)	**0.05N**	**=**	**5**
÷0.05)	**N**	**=**	**100**

f) Substitute N = 100 into equation (2)

C = 30 + 0.10N

C= 30+0.10(100)

C=30+40

C=40

Under what circumstances is Company A the better company to rent with?

The company A is better deal when the number of km (N) is greater than 100.

4. The cost to rent a movie at Video Plus is $2.00 for the first night plus $0.50 for every night after that. The cost to rent a movie at Videos-R-Us is $6.00 for 7 nights. *When is Videos-R-Us the better deal?*

a) Create a table of values for each company, let the number of nights be N and the cost be C.

Video Plus cost: C = 2 + 0.50N

N Number of Nights	**0**	**7**	**14**	**21**	**28**	**35**	**42**	**49**	**56**	**63**	**70**	**77**
C Cost ($) In dollars	**2**	**5.50**	**9.00**	**12.50**	**16.00**	**19.50**	**23.00**	**26.50**	**30.00**	**33.50**	**37.00**	**40.50**

Videos-R-Us cost: C = 6 for 7 nights

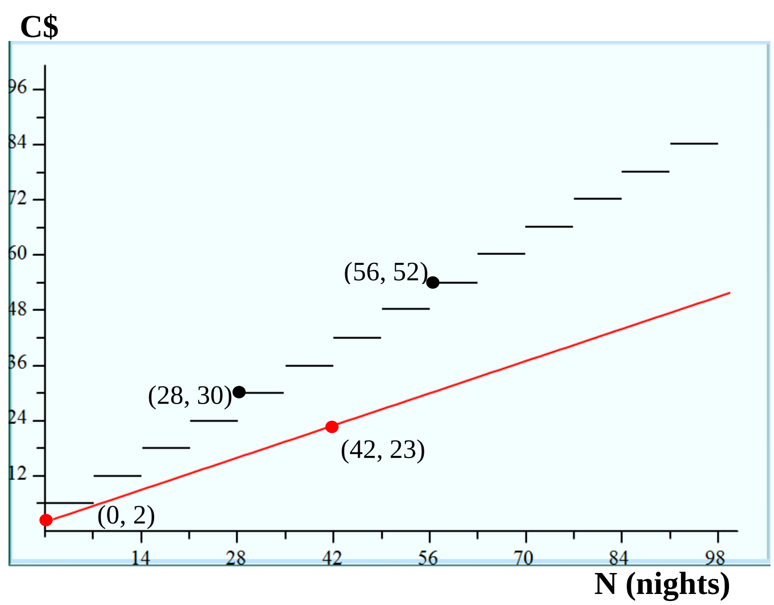
N Number of Nights	**0**	**7**	**14**	**21**	**28**	**35**	**42**	**49**	**56**	**63**	**70**	**77**
C Cost ($) In dollars	**6**	**12**	**18**	**24**	**30**	**36**	**40**	**46**	**52**	**58**	**64**	**70**

b) Create the equation for each table:

C = 2+ 0.50N (1)

C = 6 for 7 nights (2)

c) Graph each line:



d) Point of Intersection, there is no Point of Intersection.

The graph shows that there is no point of intersection.

The Video Plus is always cheaper then Videos-R-Us.

When is Videos-R-Us the better deal?

Video Plus is <u>never</u> a better deal.

5. Which method would you choose to solve the given system of equations? Why? *Justify your answer and then solve it.*

$6x + 4y = 23$ (1)

$6x + 14y = 10$ (2)

The method I would choose is the Elimination Method. The x's are easily eliminated

Solution:

	$6x + 14y$	=	10	(2)
	$6x + 4y$	=	23	(1)
(2) – (1)	$0x + 10y$	=	-13	
÷10)	y	=	-1.3	

Substitute y = -1.3 into (1)

	$6x + 4y$	=	23	(1)
	$6x + 4(-1.3)$	=	23	
	$6x - 5.2$	=	23	
+5.2)	$6x$	=	28.2	
÷6)	x	=	4.7	

The solution is (4.7, -1.3)

Check Equation (1) $6x + 4y = 23$

L.R. = $6x + 4y$ R.S = 23
= $6(4.7) + 4(-1.3)$
= $28.2 - 5.2$
= 23

Check Equation (2) $6x + 14y = 10$

L.R. = $6x + 14y$ R.S = 10
= $6(4.7) + 14(-1.3)$
= $28.2 - 18.2$
= 10

Since L.S. = R.S. for both equations, the solution x = 4.7, y = -1.3 is correct.

6. A teacher hands out a math test to 36 students. The total marks for the test is 100 and it has 38 problems. The questions are worth either 5 marks or 2 marks. *How many questions of each type of mark are on the test? Justify your answer.*

Notice: 36 students is an unnecessary piece of information.

Let x represent the number of 5 mark questions.
Let y represent the number of 2 mark questions.

The total marks for the test = 100
= 5x + 2y ∴ 5x + 2y = 100 (1)

There are 38 problems ∴ x + y = 38 (2)

Simplify and arrange terms

	5x + 2y	=	100	(1)
	x + y	=	38	(2)

Multiply equation (2) by -1 so that the x terms add to 0.

	5x + 2y	=	100	(1)
(2) × -1	-2 x - 2y	=	-76	(3)

Add equations (1) and (3)

(1) + (3)	3 x + 0y	=	24
	3 x	=	24
÷3)	x	=	8

Substitute x = 8 into equation (2) and solve for y.

	x + y	=	38	(2)
	8 + y	=	38	
-8)	y	=	30	

There are 8 five mark questions and 30 two mark questions on the test.

Check equation (1) 5x + 2y = 100

L. S. = 5x + 2y R.S. = 100
= 5(8) + 2(30)
= 40 + 60
= 100

Check equation (2) x + y = 38

L. S. = x + y R.S. = 38
= 8 + 30
= 38

Since L.S. = R.S. for both equations, the solution x = 8 , y = 30 is correct.